Synchronization of the mean velocity of a particle in stochastic ratchets with a running wave

D. E. Postnov, A. P. Nikitin, and V. S. Anishchenko
Department of Physics, Saratov State University, Astrakhanskaya Street 83, 410026 Saratov, Russia

(Received 22 July 1997; revised manuscript received 17 February 1998)

In this paper we investigate the motion of a particle in the overdamped one-dimensional system with a spatially periodic potential under the influence of a sinusoidal wave and dichotomic (binary) noise. We demonstrate the effect of synchronization between the mean velocity of a particle and the phase velocity of the running wave controlled by the noise. The results of numerical simulation are in good agreement with a full-scale experiment. [S1063-651X(98)00207-4]

PACS number(s): 05.45.+b, 05.40.+j

Recently, there has been much interest in nonlinear problems dealing with stochastic systems. In this kind of system, a variation of the parameters of external noise can qualitatively change the system's regime of operation. The well-known example is stochastic resonance. In this case, noise properties determine the type of response of a bistable system to external periodic forcing (see [1] and references therein). Another example is the so-called stochastic ratchets [2] describing nonlinear Brownian motion in a spatially periodic potential. In these systems, nonthermal fluctuations produce the directed motion of particles. The noise energy transforms into the energy of the particles' motion. The factor that defines a direction and a mean velocity of the particles' flow is fluctuation statistics and the profile of the potential function, respectively [3].

Stochastic ratchets can be considered as some sort of problem of nonlinear transport along the spatially periodic potential. Other transport mechanisms include so-called resonant activation [4], rocked ratchets [5], and particle motion by solitons [6].

The following question arises: What happens when more than one factor has an influence on the particle motion along the potential? A possible example can be found in the operation of nerve cells (neurons) where the ratchetlike chemically based transport of neurotransmitters along the axon is accompanied by running electric impulses (spikes) [7].

In the present work we focus on the behavior of the stochastic ratchet when an additional wave force is applied. The observed effect can be described as concordance of the mean velocity of the particle in stochastic ratchets with the phase velocity of the forcing wave. The characteristics of external noise play the role of control parameters.

In the stochastic ratchet, when transport takes place, the particle under transport moves randomly, but there is a nonzero drift. The mean velocity of the particle v_d,

$$v_d=\frac{1}{T}\int_0^T \dot{x}(t)dt=\frac{x(T)-x(0)}{T},$$

where T is observation time, depends on the noise intensity. We use the velocity averaged in time since it naturally follows from the task definition. Namely, we calculate to what distance a single particle can be transported in the given (long enough) time.

Let us force the stochastic ratchet by the wave with phase velocity v_ϕ. We are interested in the following question: Does the nonlinearity of the system under study exhibit the effects such as phase (frequency) locking?

We use the results obtained from a full-scale experiment for the system with phase synchronization. Under certain conditions these electronic devices demonstrate the behavior of stochastic ratchets [8]. In this case, the motion of a Brownian particle is described in terms of the mutual phase shift between two coupled oscillators. The spatial periodicity is provided by the 2π periodicity of the phase.

The mathematical model for the system in the overdamped case can be given as

$$\frac{dx}{dt}=-\frac{dV(x)}{dx}+f(x,t)+z(t)+\xi(t), \tag{1}$$

where x is a spatial variable, $\xi(t)$ is white Gaussian noise with $\langle\xi(t)\rangle=0$, $\langle\xi(t)\xi(t+\tau)\rangle=D\delta(\tau)$, D is the noise intensity, and $z(t)$ is a random process. All variables in Eq. (1) are dimensionless. We assume that $D\to 0$; hence the transport phenomena in stochastic ratchets are the result of the action of $z(t)$. $V(x)$ is an asymmetric potential with spatial periodicity, i.e., the so-called ratchet potential (Fig. 1). $f(x,t)$ describes a wave force. Let us study the simplest case when

$$f(x,t)=A\sin(\Omega t-kx). \tag{2}$$

Here the wave amplitude A is small enough to exclude the particle jumps when $z(t)=0$; Ω is the frequency and k is the wave number. The phase velocity is $v_\phi=\Omega/k$. The condition for the adiabatic approximation is

$$\tau_c \gg \tau \gg 1/\Omega, \tag{3}$$

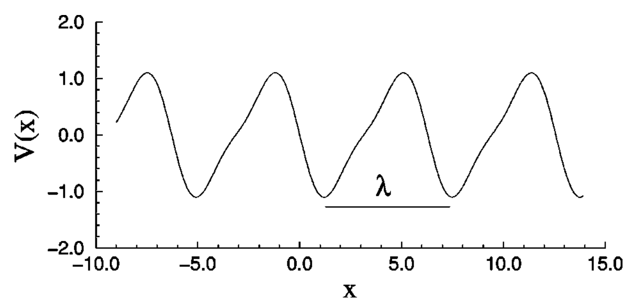

FIG. 1. Asymmetric, spatially periodic potential function $V(x)=-\sin(x)-\sin(2x)/4$.

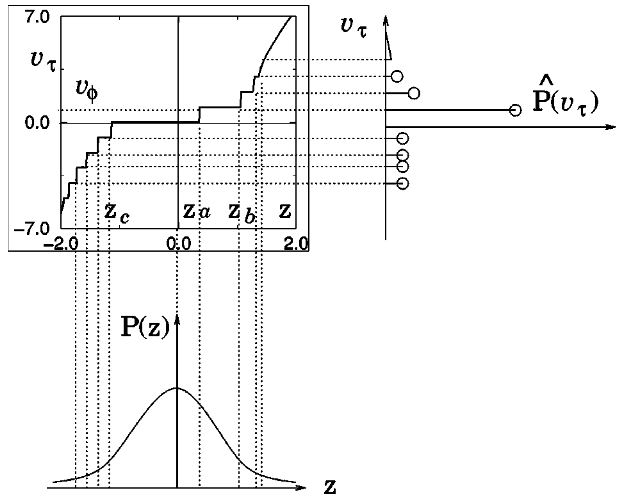

FIG. 2. Numerically obtained dependence of v_d on the z value for Eq. (1) and the rule for forming of the velocity distribution $\hat{P}(v_\tau)$ from some distribution function $P(z)$ of the applied random process.

where τ_c is the correlation time for $z(t)$ (long enough) and τ is the time interval we deal with. Assuming $z(t)=\text{const}$ during τ, one can calculate the dependence of $v_\tau=\int_0^\tau \dot{x}\,dt$ on the z value for Eq. (1). The result $v_\tau=F(z)$ is plotted in Fig. 2, where the rule for the transformation of some probability density distribution function $P(z)$ to the velocity distribution $\hat{P}(v_\tau)$ is shown as well. The dependence of v_τ on z is stairlike, which is similar to the ''Shapiro steps'' [9].

The average particle velocity now is

$$v_d=\int_{-\infty}^{+\infty} v_\tau \hat{P}(v_\tau)dv_\tau=\int_{-\infty}^{+\infty} F(z)P(z)dz. \tag{4}$$

Under the adiabatic approximation v_d is defined by $P(z)$ only via $F(z)$ for the given $V(x)$ and $f(x,t)$ in Eq. (1), while the random process $z(t)$ can be both Markovian and non-Markovian. Below we consider the case of a two-state random process with possible states Z_1 and Z_2. For the symmetric case, $Z_1=-Z_2=Z$, where Z is the noise amplitude. The corresponding probability density is given by

$$P(z)=\frac{1}{2}\delta(z-Z)+\frac{1}{2}\delta(z+Z), \tag{5}$$

where $\delta(y)$ is the Dirac delta function. According to Eq. (4),

$$v_d=\frac{1}{2}F(Z)+\frac{1}{2}F(-Z). \tag{6}$$

Let us now analyze the results from Fig. 2. For small enough noise amplitudes $Z<z_a$, $F(Z)=F(-Z)=0$ for any time and the particle does not move ($v_d=0$). For higher values $z_a<Z<z_b$ and $Z<|z_c|$, we obtain $F(-Z)=0$ and $F(Z)=v_\phi$. Thus

$$v_d=\frac{1}{2}F(Z)=\frac{1}{2}v_\phi. \tag{7}$$

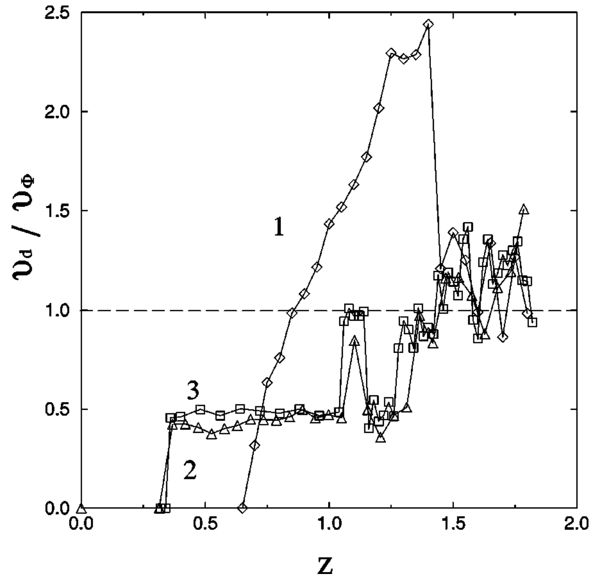

FIG. 3. Mean velocity of the particle v_d vs noise amplitude Z. (All variables are dimensionless.) 1 represents the case in which the wave forcing is absent and 2 and 3 represent the case of the wave with parameters $v_\phi=0.22$ and $A=0.5$ for the full-scale experiment and numerical simulation, respectively.

This means that for T ($T\gg\tau$), $z(t)$ switches finite times from $+Z$ to $-Z$ and from $-Z$ to $+Z$. Therefore, $\dot{x}(t)$ is a random process where the particle moves with velocity v_ϕ for half of the observation time T. When the noise amplitude Z increases further, other situations are possible. For example, if $z_b<Z<|z_c|$, one can obtain $F(-Z)=0$, $F(Z)=2v_\phi$, $v_d=v_\phi$, etc.

As $z(t)$ we use the so-called binary noise [10] (see the Appendix), which is characterized by the minimal time τ between switchings. It corresponds to the τ considered above. The results obtained are plotted in Fig. 3. In the case when the wave force vanishes v_d becomes nonzero for increasing $Z>0.75$, achieves its maximum at $Z\approx1.5$, and decreases if Z increases further (curve 1). For very large Z values the potential shape becomes negligible and $v_d\to0$ due to the symmetry of $z(t)$. The dependence described above is typical for stochastic ratchets [3]. Based on experimental results, we note that the presence of secondary peaks on curve 1 in Fig. 3 is typical for the case when $z(t)$ is binary noise. When the wave force is applied, one can see the region of $Z=0.35,\ldots,1.0$ where v_d is almost constant $v_d\approx v_\phi/2$ and the small region where $v_d\approx v_\phi$ (curves 2 and 3 in Fig. 3). This means that the results of both the full-scale experiment and numerical simulation confirm the mechanism of net current stabilization discussed above. The adiabatic condition (3) is not satisfied during the experiments, but $\tau=10$, $1/\Omega=4.5$, and $\tau>1/\Omega$. It is reasonable that the small steps in the v_d dependence were not observed for $Z>1.4$. Note that the effect we discuss is the locking of the particle in the stochastic ratchet by the running wave. At the locking, $v_d=nv_\phi$, $n=1,2,3,\ldots$. Figure 4 illustrates the locking effect when the variation of the noise amplitude Z and wave amplitude A takes place. It is easy to see the triangular zone that is typical for synchronization phenomena both in the deterministic case (Arnold tongues) and for stochastic systems [11].

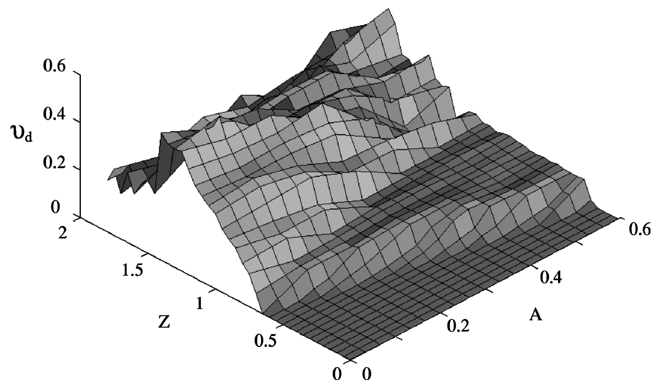

FIG. 4. Two-parametric dependence for v_d on A and Z. (All variables are dimensionless.) $v_\phi = 0.22$. The flat triangular zone corresponds to the synchronization of v_d.

Thus, in the present work we have demonstrated a nonlinear effect of wave synchronization controlled by noise. The origin of this effect is the locking of the particle motion by the running wave. To observe that effect the sufficiently long correlation time of the binary noise $z(t)$ appears to be important.

The authors would like to acknowledge Lutz Schimansky-Geier and Fabio Marchesoni for helpful discussions and critical comments. This work was supported by a common research project of DFG and RFRF [Grant No. 436 RUS 113/334/0(R)].

APPENDIX: DEFINITION OF THE BINARY NOISE

The binary noise $z(t)$ is a non-Markovian two-state random process when the switchings from one state to another occur in discrete numbers of time moments. This is a more realistic version of dichotomous noise. Any information signal in the binary code used by computers or other communication devices can be considered as an example. For the symmetric case two possible states are $+Z$ and $-Z$. Assuming that the first observed switching occurs at $t_0 = \Delta$, any other switchings can take place at discrete numbers of moments t_s,

$$t_s = k\tau + \Delta, \quad k = 0,1,2,3,\ldots. \tag{A1}$$

It is assumed that Δ is the random value uniformly distributed within $[0,\tau]$; τ is the time scale of binary noise, i.e., the shortest possible time between two switchings. The probability density reads

$$p(z,t) = P_+(t)\,\delta(z-Z) + P_-(t)\,\delta(z+Z), \tag{A2}$$

where probabilities $P_\pm(t)$ satisfy

$$\frac{d}{dt}\begin{pmatrix} P_+ \\ P_- \end{pmatrix} = \begin{pmatrix} -\gamma & \gamma \\ \gamma & -\gamma \end{pmatrix}\begin{pmatrix} P_+ \\ P_- \end{pmatrix} \tag{A3}$$

and

$$\gamma = \frac{1}{2}\sum_{k=0}^{\infty} \delta(t - k\tau - \Delta).$$

[1] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14**, 453 (1981); B. McNamara and K. Wiesenfeld, Phys. Rev. A **39**, 4854 (1989); F. Moss, *Stochastic Resonance: From the Ice Ages to the Monkey Ear—Some Problems in Statistical Physics*, edited by G. Weiss (SIAM, Philadelphia, 1992); L. Gammaitoni, P. Jung, P. Hänggi, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).

[2] M. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993); **72**, 2656 (1994); J. Prost, J.-F. Chauwin, L. Peliti, and A. Ajdari, *ibid.* **72**, 2652 (1994); S. M. Simon, C. S. Peskin, and G. F. Oster, Proc. Natl. Acad. Sci. USA **89**, 3770 (1992); W. Sung and P. J. Park, Phys. Rev. Lett. **77**, 783 (1996); J. Rousselet, L. Salome, A. Ajdari, and J. Prost, Nature (London) **370**, 446 (1994); L. P. Faucheux, L. S. Bourdieu, P. D. Kaplan, and A. J. Libchaber, Phys. Rev. Lett. **74**, 1504 (1995); R. D. Astumain, Science **276**, 917 (1997).

[3] Ch. R. Doering, W. Horsthemke, and J. Riordan, Phys. Rev. Lett. **72**, 2984 (1994); M. Bier, Phys. Lett. A **211**, 12 (1996); M. M. Millonas and M. I. Dykman, *ibid.* **185**, 65 (1994); R. D. Astumian and M. Bier, Phys. Rev. Lett. **72**, 1766 (1994); Ch. R. Doering, Nuovo Cimento D **17**, 685 (1995); R. Bartussek, P. Hänggi, B. Lindner, and L. Schimansky-Geier, Physica D **109**, 17 (1997); P. Reimann, R. Bartussek, R. Häussler, and P. Hänggi, Phys. Lett. A **215**, 26 (1996); R. Bartussek, P. Reimann, and P. Hänggi, Phys. Rev. Lett. **76**, 1166 (1996); J. Luczka, R. Bartussek, and P. Hänggi, Europhys. Lett. **31**, 431 (1995).

[4] P. Jung, Phys. Rep. **234**, 175 (1993).

[5] R. Bartussek, P. Hänggi, and J. G. Kissner, Europhys. Lett. **28**, 459 (1994); I. Zapata, R. Bartussek, F. Sols, and P. Hänggi, Phys. Rev. Lett. **77**, 2292 (1996); T. E. Dialynas, Katja Lindenberg, and G. P. Tsironis, Phys. Rev. E **56**, 3976 (1997).

[6] F. Marchesoni, Phys. Rev. Lett. **77**, 2364 (1996); A. Savin, G. P. Tsirons, and A.V. Zolotoryuk, Phys. Lett. A **229**, 279 (1997).

[7] M. R. Rosenzweig, A. L. Leiman, and S. M. Breedlove, *Biological Psychology* (Sinauer, Sunderland, MA, 1996).

[8] D. E. Postnov, A. P. Nikitin, and V. S. Anishchenko, Tech. Phys. Lett. **22**, 352 (1996); A. P. Nikitin and D. E. Postnov, *ibid.* **24**, 61 (1998); D. E. Postnov, A. P. Nikitin, A. G. Balanov, and V. S. Anishchenko, *Book of Abstracts of the International Conference on Nonlinear Dynamics and Chaos, Saratov, Russia, 1996*, edited by V. S. Anishchenko and G. I. Strelkova (College, Saratov, 1996), p. 195.

[9] S. Shapiro, A. R. Janus, and S. Holly, Rev. Mod. Phys. **36**, 223 (1964); P. Jung and P. Hänggi, Ber. Bunsenges. Phys. Chem. **95**, 311 (1995).

[10] V. I. Tikhonov and M. A. Mironov, *Markov Processes* (Sovier Radio, Moscow, 1977).

[11] B. V. Shulgin, A. B. Neiman, and V. S. Anishchenko, Phys. Rev. Lett. **75**, 4157 (1995).